Two-Dimensional Computer Experiments on Ion-Beam Neutralization

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The two-dimensional electron-ion mixing mechanism for ion propulsion has been programed into a computer. Electrons and ions are simulated by several thousand rods of negative and positive charge. They are accelerated step-by-step, the space-charge fields being evaluated at each step by a new superfast technique of integrating Poisson's equation. The technique employs Fourier analysis, a marching method, and a capacitance matrix characterizing the electrode system. Integration takes approximately one second for 2% linear resolution. A stack of strip ion beams is injected through an accel-grid. Thermal electrons are released from both sides into each beam. The electron emitters define the decel-potential but are not placed directly within the beams; free space conditions are imposed at the heads of the advancing beams. Trajectory and equipotential plots show that the electron supply and demand of the beams is regulated by fluctuating fields. A near-neutral plasma is formed at a potential within a few kT of the electron emitters. The thrust is thus maintained near a value corresponding in ion acceleration to this potential, beam spread remaining negligible. Changing parameters, such as masses, currents, and velocities produce expected results, such as ion turn around when electron emission is reduced to zero value.

Introduction

At the 1963 AIAA Electric Propulsion Conference the simulation of neutralizer operation in an ion engine, on a high-speed digital computer, was reported.[1] At that time only a one-dimensional model had been programed. We present in this paper the computer simulation of neutralizer operation in two dimensions and the successful achievement of neutralization in this computer model.

It should be recalled that the process of neutralization presents a challenging problem, because of the following conditions:

1) Collisions cannot be relied upon to provide electron-ion mixing.

2) The random velocities of the electrons emitted by the neutralizer exceed the directed velocity of the ions,[2] and somehow the electrons must be "tamed" to go along with the ions.

3) No satisfactory d.c. state has been found in which neutralization is achieved. Where such states have seemed ideally possible, they have been found to be unstable.[3]

4) Theoretical analyses by conventional methods are restricted to one dimension.

One-Dimensional Studies

A complete account of strictly static configurations in one-dimension was presented by Derfler,[3] and his somewhat negative results agree with the observations in the one-dimensional computer studies: a steady state is not achieved but instabilities result in fluctuations that produce neutrality-in-the-mean. The one-dimensional computer model was thus able to give reassuring answers regarding neutralization. In that model an infinite plane and fine mesh of electron-emitting wires had been immersed across an infinitely wide ion beam. Self-excited fluctuating space-charge fields provided the mixing and led to the creation of a good streaming plasma, yielding near-ideal thrust, although the primary electron emission needed for such operation was found to be rather high. The fluctuations occurred at the electron plasma frequency and reached potential amplitudes substantially exceeding the kT level. These fluctuations regulated the supply and demand of electrons (unwanted electrons being reabsorbed into the wire mesh), and were presumably excited by the predicted instability.

In one sense the concept of a "static" configuration ought to be broadened to cover the case of an advancing head of the beam. One-dimensional computer studies show that a large fraction of the electrons proceed to the head of the beam and then bounce back because of the potential barrier set up by the ions. The advancing boundary thus takes part in the mixing process, and one wonders whether a pseudostatic configuration of an ever lengthening potential trough for the electrons can, perhaps, give good mixing without high-frequency fluctuations. Whereas no analytical studies of this model are available, computer studies show that both fluctuations and reflections are important.

Two-Dimensional Model

In going from one to two dimensions, it can be argued that successful neutralization becomes less likely because the electrons are not injected across the entire width of an ideal infinitely broad ion beam. On the other hand, lateral beam boundaries are present in the more realistic two-dimensional geometry and mixing by reflection from potential walls should improve. (The orbit of an electron that bounces elastically within a rectangular box should, according to the ergodic principle, eventually cover the interior densely.) Which of the two conflicting tendencies in the transitions from one to two dimensions will dominate can only be answered by two-dimensional computer studies.

Again the question of a pseudostatic configuration, with mixing entirely by reflection rather than fluctuation, arises. Purely analytical methods are not available to solve even

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Presented as Preprint 64-698 at the AIAA Fourth Electric Propulsion Conference, Philadelphia, Pa., August 31–September 2, 1964; revision received January 15, 1965. This work was done under NASA Contract No. NAS-3-2503. The authors gratefully acknowledge the contributions of S. P. Yu and R. Hockney. Helpful comments by one of the reviewers, J. M. Sellen Jr., which led to additions to the paper, are gratefully acknowledged.

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strictly static two-dimensional charge flow problems and all studies of this kind rely heavily on computations. However, rather than utilize one of the many conventional computing programs for calculating possible static configurations we employ here, as before, a method that does not anticipate the existence of a static state and that will exhibit time-varying phenomena, should they develop and persist. This philosophy has paid good dividends in the study of microwave tube operation.

We consider the two-dimensional neutralizer geometry shown in Fig. 1. It is a compromise between a realistic geometry (such as that used in actual experiments by Sellen et al.,[4] Brewer,[5] and Eilenberg et al.[19] and demands on electrode configuration dictated by computer economy. A stack of parallel-strip ion beams is injected across the accel-grid while injection of thermal electrons proceeds from hot electrodes into both sides of each ion beam. The hot electrodes serve to define the decel-potential (see the schematic potential profile in Fig. 1), preventing electrons from reaching the ion source, and forcing them downstream instead.

In some earlier runs we specified uniform potential along the entire decel-plane. It implied that there existed a fine mesh of grid wires across the entire beam (indicated by dots in Fig. 1). However, electrons were emitted only from limited sections of the decel-plane. This case is referred to as the "closed grid" case. Later, we removed the ideal grid wires from the decel-plane, leaving only the hot emitting plates in what we call the "open grid" geometry.

Our computer simulation of this system treats the charged particles as rods. For computer memory, economy, and speed, many actual particles are lumped into one rod. Typically we would handle several thousand rods, positively or negatively charged, and a unit length of each rod might stand for some 10^7 ions or electrons. The effect of this coarse graining is found negligible in experiments with different grain sizes. The motion of each rod is traced stepwise through the time-varying fields, in simulation of the transient development of the beam. Initially the system is void of either ion or electron rods. The fields are evaluated at every step from the instantaneous space-charge distribution.

The entire evaluation of the fields takes about two seconds. The transient formation of the beam could thus be traced in steps, which represent a small fraction of a plasma oscillation cycle, by an IBM 7094 in about one hour.

Ion rods are injected at a constant rate through the limited areas of the accel-plane, as shown in Fig. 1. Electron rods are injected with random velocity components according to the half-Maxwellian distribution, both from the upstream and the downstream faces of the hot decel-electrodes.

Evaluation of Space-Charge Fields

Whereas the step-by-step advancement of several thousand particles from given fields is a moderately fast procedure, the evaluation of the instantaneous two-dimensional field components from the given charge distribution presents difficulties. Well established iterative and/or relaxation procedures had to be discarded as too slow.

A new direct procedure using Fourier analysis, recently developed by Hockney and Buneman,[6,7] was employed. The half-period of our geometry is subdivided into a mesh of 24 by 200 mesh points. This mesh size is smaller than a Debye length and hence fine enough to resolve the formation of "sheaths" and virtual cathodes. The system is periodic in the direction of the 24 mesh points, and we employ a 24-point Fourier analysis. In the beam direction we employ, for each Fourier component, a "marching method"[8,9] of integrating Poisson's equation.

At the accel-plane the potential is prescribed, whereas beyond the head of the beam (out in space) we specify the genuine free space condition that all field components should vanish. The potential at the decel-system is pulled up to the specified level at the electrodes by means of a superposition procedure. The capacitance matrix[9] for the mesh points identifying the hot electrodes is calculated and stored. From it the electrode surface charges are obtained at each step, to be included with the free ion and electron charges in our procedure for solving Poisson's equation, in order to raise the electrode potential to the specified level. Actually, part of our integration had to be run twice; first without the surface charges, to determine how much potential correction is needed, and then again with the surface charges. The second integration increased the computer time by only a small fraction.

Adjustable Parameters

The dynamic equations of the system are programed with the variables and parameters in normalized form. The various parameters are as follows:

L = distance between ion emitter and accel-grid
l = distance between accel-grid and decel-emitter plane
M = ion mass
m = electron mass
φ_1 = potential between accel-grid and decel-emitter
φ_0 = potential between ion emitter and accel-grid
kT = mean electron emission energy (two degrees of freedom)
$e\varphi_0$ = ion kinetic energy at accel-grid
N_i = number of ions injected per unit time step
N_e = number of electrons injected per unit time step
N = number of ions in the spacing between accel- and decel-planes when drifting at injection velocity
$2a$ = ion beam width at injection plane
ω_p = electron plasma radian frequency
Δt = value of the unit time step

Results

Figure 2 shows typical ion trajectories for the open grid case, plotted in the x-y domain with time as the running parameter along the trajectories. The distance unit and mesh size is $l/8$ in all runs. There is a residual ion-beam spread due to the low value of ion-to-electron mass ratio (144 for this case). The ion-beam spread is found to be inversely proportional to ion mass, and would be invisible for realistic values of ion mass; small values of ion mass are used for computer economy. Apparent reflection at the top repre-

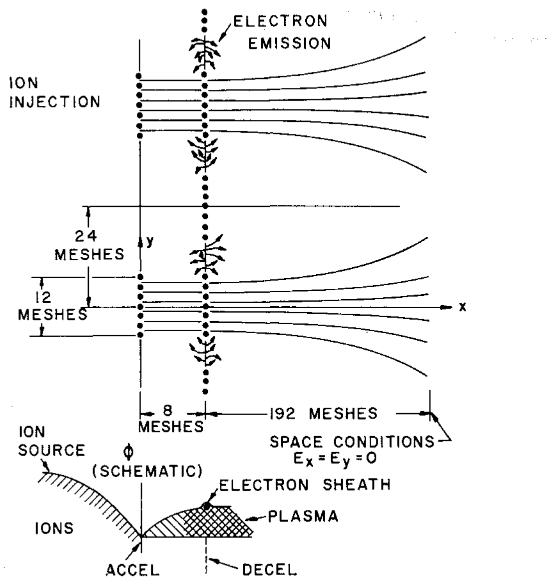

Fig. 1 A schematic of electron and ion injection in an ion engine.

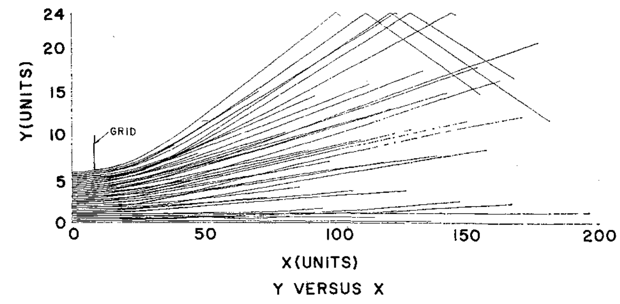

Fig. 2 Ion trajectories, Y vs X; M/m = 144, l/L = 1.5, N_i = 3, N = 90, a/l = 0.75, φ_1/φ_0 = 0.4, N_e/N_i = 3, $\omega_p\Delta t$ = 0.28222, and $e\varphi_0/kT$ = 14.4.

sents ions moving from the neighboring cell. For the case of the closed grid, the ion-beam spread is slightly greater. This is due to the difference in the potential distribution in the two cases.

Figure 3 shows typical electron trajectories for the open grid case in the x-y domain. Electrons follow ions very closely. For the case of an ideal grid (closed-grid case), the complete grid becomes a virtual emitter, even though electrons are emitted over only a small portion of the decel-grid. This is again due to the difference in nature of the potential distribution in the two cases. Because of the decel-potential in the system, electrons penetrate upstream, but do not reach the accel-grid; the penetration of electrons toward the accel-grid is determined by the beam-plasma junction conditions.[9]

Figure 4 shows the equipotentials in the x-y domain after a few time steps, when there are very few charged particles. It is apparent that the penetration of negative potential through the wider openings in the decel-system forces the electrons toward the ion beam; this results in faster space-charge neutralization. Figure 5 shows the equipotentials in the x-y domain at the 600th time interval when the system is fluctuating about a steady mean. The presence of good plasma is indicated by the potential plateau.

Figure 6 shows sampled electron trajectories in the x-time domain. It is evident that electrons oscillate back and forth in space. High velocity electrons turn around. These fluctuations occur very nearly at the electron-plasma frequency. In fact, the total number of electrons minus ions in space also fluctuates at the electron-plasma frequency for a constant rate of ion and electron injection.[20] In general, more electrons are emitted than ions. Unwanted electrons are absorbed on the decel-electrode after having made short excursions into the negative "sheath" that develops around the emitter. Often these electrons do not get out of the sheath, but a few electrons return after a long excursion into the plasma. These electrons are deleted from the computer storage. Practically, a virtual cathode exists at the outer edge of the negative sheath from which only the wanted electron current escapes. The mesh size in our calculations is fine enough to

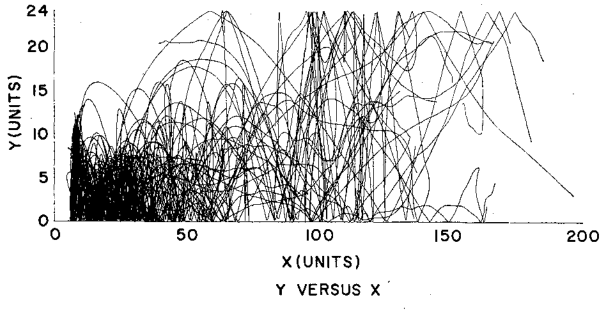

Fig. 3 Electron trajectories, Y vs X; M/m = 144, l/L = 1.5, N_i = 3, N = 90, a/l = 0.75, φ_1/φ_0 = 0.4, N_e/N_i = 3, ω_p Δt = 0.28222, and $e\varphi_0/kT$ = 14.4.

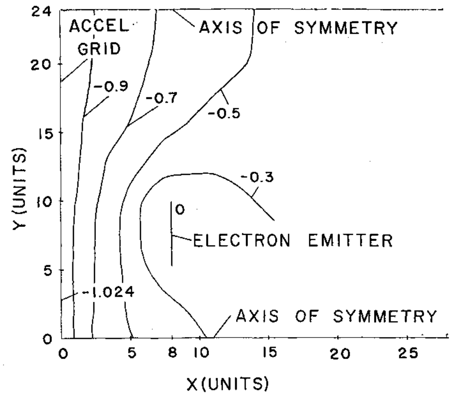

Fig. 4 Normalized equipotentials in normalized space; time step: 10 M/m = 144, l/L = 1.5, N_i = 3, N = 90, a/l = 0.75, φ_1/φ_0 = 0.4, N_e/N_i = 3, $\omega_p\Delta t$ = 0.28222, and $e\varphi_0/kT$ = 14.4.

reproduce the formation of sheath and virtual cathode in detail as a time-varying process: evidently, the fluctuations have regulated the supply and demand of electrons.

The previous figures indicate the results of a practical case when a sufficient number of electrons is emitted for space-charge neutralization of the beam. Figure 7 shows typical ion trajectories in the x-time domain for the case of zero electron emission. Most of the ions are turned around. The ion bunches are associated with the ion-plasma frequency. Similar results have been obtained by Lomax,[10] Dunn and Ho,[11] and Birdsall and Bridges[12] in studies of electron dynamics.

The efficiency of neutralization depends upon the electron emission from the hot electrode. The minimum electron current needed for adequate space-charge neutralization is found to be less critical in the two-dimensional model than in the one-dimensional model. We believe that this is due to the additional degree of freedom for the electrons.

Figure 8 shows the variation of ship potential with time for three different values of mass ratio; ship potential φ is defined as the potential difference between the decel-electrode and "infinity." It is noticed that the potential fluctuations are reduced as the ion mass is increased, implying thereby that the ship-potential fluctuations would be small for the realistic case of heavy ions. For computer-technical reasons, the recorded potential values become questionable beyond time steps 600–700 approximately.

Figure 9 shows the variations of the ship potential with time for three different values of electron emission. For the case of N_e/N_i = 0, the ship becomes negatively charged as ions leave the ship. Fluctuations are increased as the electron emission

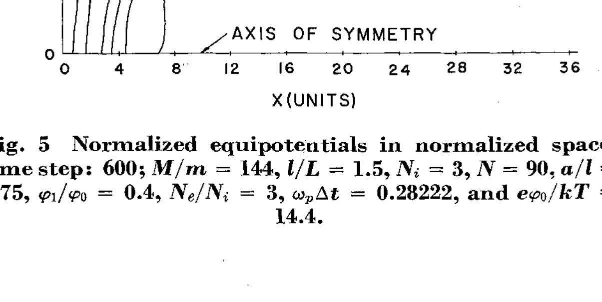

Fig. 5 Normalized equipotentials in normalized space; time step: 600; M/m = 144, l/L = 1.5, N_i = 3, N = 90, a/l = 0.75, φ_1/φ_0 = 0.4, N_e/N_i = 3, $\omega_p\Delta t$ = 0.28222, and $e\varphi_0/kT$ = 14.4.

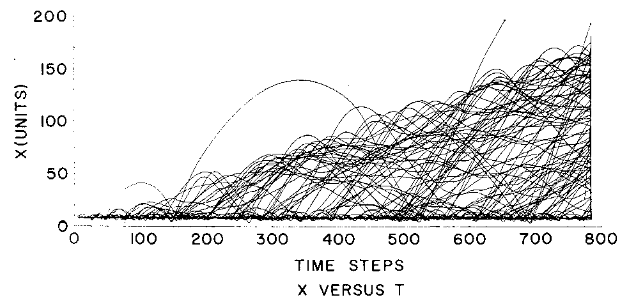

Fig. 6 Electron trajectories, X vs T; $M/m = 256$, $l/L = 1.125$, $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, $N_e/N_i = 4$, $\omega_p \Delta t = 0.2822$, and $e\varphi_0/kT = 25.6$.

is increased beyond an optimum value. Plots of normalized thrust are shown in Figs. 10 and 11. In Fig. 10 comparison is made for different mass ratios; in Fig. 11 for different emission ratios. It is noticed that the fluctuations are reduced as the ion mass is increased, and thrust deteriorates in the case of an inadequate supply of electrons. For the case of $N_e/N_i = 2$, the results are similar to those for the case of $N_e/N_i = 4$. For $N_e/N_i = 0$ the thrust goes to zero average eventually, but full records of all the contributions to the thrust figure are only available up to 200 time steps. The open and closed-grid cases yield almost identical results.

Discussion

A. Comparison with Physical Experiments

Broadly speaking, the computed results confirm what experimental evidence[4] on neutralization is available. They show, as had been observed in laboratory tests,[4] and as had been hoped for space conditions, that neutralization does occur when an adequate supply of electrons is made available near the ion beam. The July 1964 space tests on the 2500-mile suborbital space electric rocket (SERT) engine have confirmed that the removal of boundaries to infinity does not impair the neutralization process. This result agrees with the prediction of the computations. Boundary conditions as they prevail in free space were programed into the computer. In regard to details of comparison (quantitative) between computation and experiment, it must be kept in mind that the results given in this paper are for the simulation of an ion engine with rather crude and, in some respects, unrealistic boundary conditions. Hence comparison with actual experiments cannot be expected to be perfect. To enumerate some of these limitations:

1) A periodicity in the transverse direction is assumed, merely for computer economy in solving Poisson's equation. When the beam is confined in the central region of a cell the interaction from the neighboring cells is negligible.

2) The computer runs are initiated with empty space and carried out to time intervals limited by the available computer memory. Particles leaving the region simulated in the program are delected from the program.

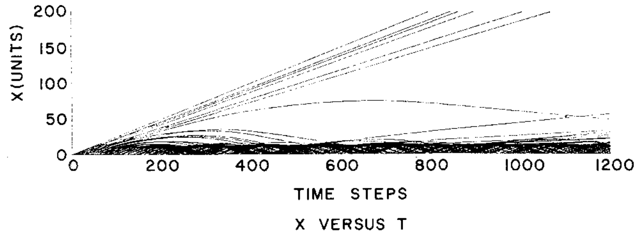

Fig. 7 Ion trajectories, X vs T; $M/m = 144$, $l/L = 1.5$, $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, $N_e/N_i = 0$, and $\omega_p \Delta t = 0.2822$.

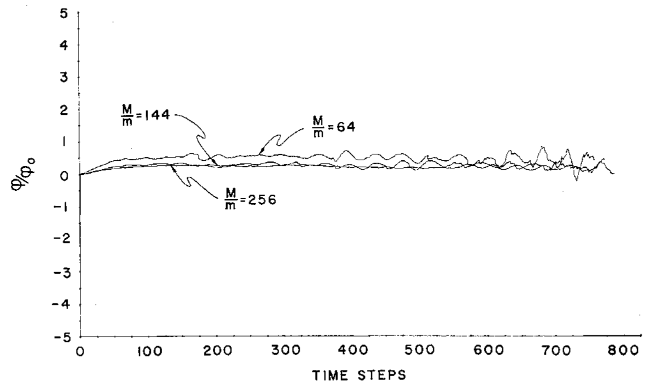

Fig. 8 Ship potential vs time; variation with mass; $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, $N_e/N_i = 4$, and $\omega_p \Delta t = 0.2822$.

3) For purposes of computer economy the x and y coordinates of each particle are stored in one computer word by appropriate packaging. This necessitates the use of small values of ion-to-electron-mass ratio, and extrapolation to the actual values (cesium or mercury ions) is made from comparative studies at three different, but still small, values of the mass ratio.

4) In the computations a properly collimated beam is introduced into the neutralizer region across an ideal rigid plane boundary between the neutralizer and the ion gun. No interplay between plasma formation on one side of the boundary and beam focusing effects on the other side (the gun region) has been taken into account.

5) Complete vacuum is assumed; this implies absence of neutral particles, and hence, charge exchange is neglected.

6) Binary collisions between charged particles are not programed because during the time intervals and within the space covered by the calculations such events are rather improbable.

7) The analysis is made strictly in a two-dimensional configuration in which rods of infinite length (perpendicular to the plane of the paper) are used for charged particles.

8) The emitter was taken to be a plane surface emitting on both sides (again for reasons of computer convenience), and only one sphcific choice of location and shape of the emitter in relation to the beam is dealt with here. Small circular emitters in various positions are being programed in further studies.

B. Beam Divergence

The extrapolation of the beam divergence is based upon linear inverse proportionality to ion mass. The transverse periodicity, not realized in some physical experiments, tends

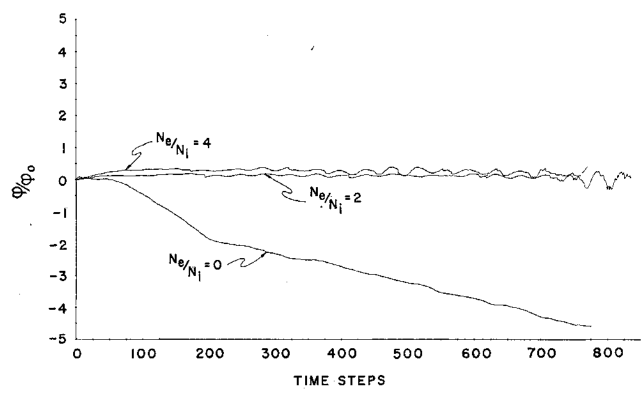

Fig. 9 Ship potential vs time; variation with electron emission; $M/m = 144$, $l/L = 1.5$, $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, and $\omega_p \Delta t = 0.2822$.

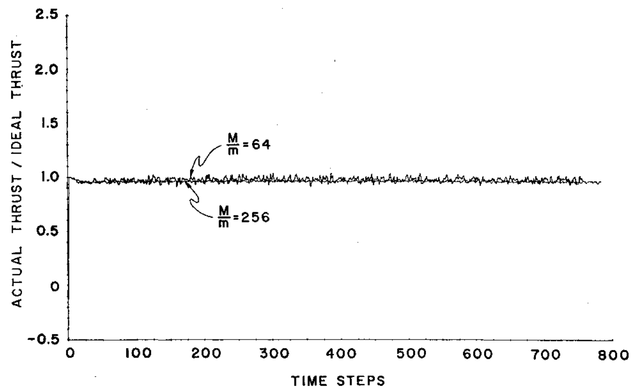

Fig. 10 Thrust vs time; variation with mass; $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, $N_e/N_i = 4$, $\omega_p\Delta t = 0.2822$.

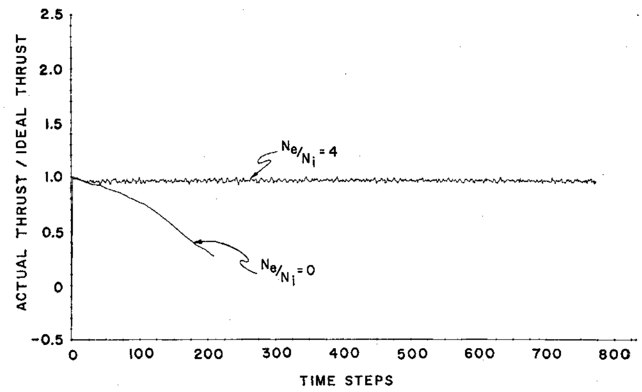

Fig. 11 Thrust vs time; variation with electron emission; $M/m = 144$, $l/L = 1.5$, $N_i = 3$, $N = 90$, $a/l = 0.75$, $\varphi_1/\varphi_0 = 0.4$, $\omega_p\Delta t = 0.2822$.

to reduce the ion-beam divergence. A slight reduction in the primary electron emission (even under the conditions of electron sheath formation around the electron emitter) increases the ion-beam divergence. Aperture effects in an actual system will result in a finite ion-beam divergence[13] even with complete or near-complete neutralization.

C. Fluctuations

According to the computations, self-excited fluctuating space-charge fields help to provide the mixing and lead to the creation of a good streaming plasma. The fluctuation level exceeds the thermal level by a substantial factor and is of the order of several tens of volts in a realistic ion beam of several kilovolts. Fluctuations of this kind help, in a way, to "stabilize" the plasma. These fluctuations are longitudinal and therefore difficult to pick up. A conversion to electromagnetic waves is usually a second-order effect and does not readily occur at moderate amplitudes. Some localized oscillations in the region of the emitter are reported by Bernstein and Sellen,[13] but not at an exceptionally high level.

Further down the beam, at distances well beyond those covered by the computations, Bernstein and Sellen[13] report exceptionally low levels of fluctuations, in the form of "millivolt" electron temperatures. One possible explanation of such a quiet plasma is that the accelerated ion beam, in a frame traveling at its exit speed, appears as an extremely "cold" assembly of particles, and the electrons, again by collective mechanisms, cool down against the ions.

The presence of fluctuations, and their role in the mixing process, should certainly be further investigated by physical measurements. Whereas neutralization in a quasi-static configuration due to reflections from boundaries alone cannot be eliminated in principle, the computations indicate an almost turbulent state and greater randomness than static conditions would permit. (Notice, for instance, that in static conditions all particles would retain their total energy and also that distinct patterns and "caustics" of particle flow would appear.)

Fluctuations at a high level are due to some instability such as that discussed by Derfler.[3] The saturation of such instabilities to a level which results in "average" stability has been analyzed theoretically in one dimension by Drummond and Pines,[14] and Vedenov, Velikhov, and Sagdeev.[15] An instability due to a cold ion beam passing through a thermal distribution of electrons, may serve to cause the necessary entropy increase as discussed by Buneman[16] and thus perhaps provide a well mixed beam in thermodynamic equilibrium.

Some strong randomizing element is necessary to explain the readiness with which plasmas are formed and brought into stabilized equilibrium. Given a randomizing agent somewhere, for instance at a midplane as artificially postulated in the calculations by Sellen et al.,[17] mixing and neutralization can be readily understood. The computations seem to have shown up a "deus ex machina" in the form of self-created fluctuations; their physical occurrence has certainly been confirmed by some plasma experiments, such as those of Garscadden and Emeleus.[18]

Computer time and memory limitations prevented use of times longer than several ion-plasma periods. There are, in a plasma without magnetic field, no oscillations, resonances, or instabilities at ion-plasma frequency. However, the computations are sensitive to "ion waves" that propagate at velocity $(kT_e/m_i)^{1/2}$. These are among the fluctuations stimulated by the streaming situation and are also contained in the instability analysis given by Derfler.[3]

Conclusions

A fast method for analyzing the space-charge neutralization mechanism in a two-dimensional ion engine configuration has been presented. This technique is a powerful and practical method for studying two-dimensional space-charge problems with arbitrary boundary constraints. It has been applied here to the problem of creating a moving plasma. This is of interest not only in ion propulsion, but also in the generation of neutral beams for other applications.

A good plasma is formed and the ion beam remains well compressed when enough electrons are emitted. Residual ion-beam spread is found to be inversely proportional to ion mass and would be invisible for heavy (cesium or mercury) ions. Cutting off the electron emission leads to ion turn-around and thrust deterioration. Ship-potential fluctuations of varying intensity seem to be present in all cases.

Neutralization from the side of an ion beam is apparently no problem. It occurs readily when more than sufficient electrons are made available. This results in the formation of a moderately steady moving plasma.

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